# Assignment

## System of Co-ordinates

### Basic Level

1. The distance between the points $(17, 105^o)$ and $(5\sqrt{2}, 60^o)$ is

(a) 13 (b) 12 (c) 11 (d) 10

2. In a plane, the co-ordinates $(r, \theta)$ of a point are equivalent

(a) $(r, -\theta)$ (b) $(-r, \theta)$ (c) $(-r, \pi + \theta)$ (d) $(r, \pi + \theta)$

3. The system of coordinates known as the cartesian system of coordinates was first introduced by

(a) Euclid (b) Euler (c) Descarte (d) Bhasker

4. Which of the following polar coordinates are associated to the same point

I : $(2, 30^o)$ II : $(3, 150^o)$

III : $(-2, 45^o)$ IV : $(-3, 330^o)$

V : $(3, -210^o)$ VI : $(-3, 30^o)$

(a) I, III and IV (b) II, IV and VI (c) II, IV, V and VI (d) IV and VI

## Distance Formula

### Basic Level

5. If the distance between the points ($a$, 2) and (3, 4) be 8, then $a$ = [MNR 1978]

(a) $2 + 3\sqrt{15}$ (b) $2 - 3\sqrt{15}$ (c) $2 \pm 3\sqrt{15}$ (d) $3 \pm 2\sqrt{15}$

6. The distance between the points $(am_1^2, 2am_1)$ and $(am_2^2, 2am_2)$ is

(a) $a(m_1 - m_2)\sqrt{(m_1 + m_2)^2 + 4}$ (b) $(m_1 - m_2)\sqrt{(m_1 + m_2)^2 + 4}$

(c) $a(m_1 - m_2)\sqrt{(m_1 + m_2)^2 - 4}$ (d) $(m_1 - m_2)\sqrt{(m_1 + m_2)^2 - 4}$

7. The distance of the point $(b\cos\theta, b\sin\theta)$ from origin is [MP PET 1984]

(a) $b\cot\theta$ (b) $b$ (c) $b\tan\theta$ (d) $b\sqrt{2}$

8. The distance between the points $(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta,\ a\sin\beta)$ is

(a) $a\cos\frac{\alpha-\beta}{2}$ (b) $2a\cos\frac{\alpha-\beta}{2}$ (c) $a\sin\frac{\alpha-\beta}{2}$ (d) $2a\sin\frac{\alpha-\beta}{2}$

9. The point on $y$-axis equidistant from the points (3, 2) and (–1, 3) is

(a) (0, –3) (b) $(0, -3/2)$ (c) $(0, 3/2)$ (d) (0, 3)

10. The point $P$ is equidistant from $A$(1, 3), $B$(– 3, 5) and $C$(5, –1). Then $PA$ = [EAMCET 2003]

(a) 5 (b) $5\sqrt{5}$ (c) 25 (d) $5\sqrt{10}$

11. The point whose abscissa is equal to its ordinate and which is equidistant from the points (1, 0) and (0, 3) is

(a) (1, 1) (b) (2, 2) (c) (3, 3) (d) (4, 4)

12. Mid-point of the sides $AB$ and $AC$ of a $\Delta ABC$ are (3, 5) and (–3, –3) respectively, then the length of the side $BC$ is

(a) 10 (b) 20 (c) 15 (d) 30

13. The distance of the middle point of the line joining the points $(a\sin\theta, 0)$ and $(0, a\cos\theta)$ from the origin

(a) $\frac{a}{2}$ (b) $\frac{1}{2}a(\sin\theta+\cos\theta)$ (c) $a(\sin\theta+\cos\theta)$ (d) $a$

14. A point on the line $y = x$ at a distance of 2 units from the origin is [MP PET 1984]

(a) $(0, \sqrt{2})$ (b) $(\sqrt{2}, 0)$ (c) $(2, 2)$ (d) $(\sqrt{2}, \sqrt{2})$

15. If the points (1, 1), (–1, –1) and $(-\sqrt{3}, k)$ are vertices of an equilateral triangle then the value of $k$ will be

(a) 1 (b) –1 (c) $\sqrt{3}$ (d) $-\sqrt{3}$

## Advance Level

16. If $O$ be the origin and if the coordinates of any two points $Q_1$ and $Q_2$ be $(x_1, y_1)$ and $(x_2, y_2)$ respectively, then $OQ_1.OQ_2\cos Q_1OQ_2 =$ [IIT 1961]

(a) $x_1x_2 - y_1y_2$ (b) $x_1y_1 - x_2y_2$ (c) $x_1x_2 + y_1y_2$ (d) $x_1y_1 + x_2y_2$

17. If the line segment joining the points $A(a, b)$ and $B(c, d)$ subtends an angle $\theta$ at the origin, then $\cos\theta$ is equal to [IIT 1961]

(a) $\frac{ab+cd}{\sqrt{(a^2+b^2)(c^2+d^2)}}$ (b) $\frac{ac+bd}{\sqrt{(a^2+b^2)(c^2+d^2)}}$ (c) $\frac{ac-bd}{\sqrt{(a^2+b^2)(c^2+d^2)}}$ (d) None of these

18. The vertices of a triangle $ABC$ are (0, 0), (2, –1) and (9, 2) respectively, then $\cos B =$ [AMU 1977]

(a) $\frac{11}{290}$ (b) $\frac{\sqrt{11}}{290}$ (c) $-\frac{11}{\sqrt{290}}$ (d) $-\sqrt{\frac{11}{290}}$

19. If $A(2, 2), B(-4, -4), C(5, -8)$ are vertices of any triangle, then the length of median passes through $C$ will be [Rajasthan PET 1988]

(a) $\sqrt{65}$ (b) $\sqrt{117}$ (c) $\sqrt{85}$ (d) $\sqrt{113}$

20. If a vertex of an equilateral triangle is on origin and second vertex is (4, 0), then its third vertex is

(a) $(2, \pm\sqrt{3})$ (b) $(3, \pm\sqrt{2})$ (c) $(2, \pm 2\sqrt{3})$ (d) $(3, \pm 2\sqrt{2})$

21. The locus of the point *P* equidistant from the points $(x_1, y_1)$ and $(x_2, y_2)$ is $(x_1 - x_2)x + (y_1 - y_2)y + c = 0$, then the value of *c* is

(a) $(x_1^2 - x_2^2) + (y_1^2 - y_2^2)$ (b) $\frac{1}{2}(x_1^2 + x_2^2 + y_1^2 + y_2^2)$ (c) $\frac{1}{2}(x_2^2 - x_1^2 + y_2^2 - y_1^2)$ (d) $\sqrt{x_1^2 - x_2^2 + y_1^2 - y_2^2}$

22. Let $S_1, S_2, ....$ be squares such that for each $n \geq 1$, the length of a side of $S_n$ equals the length of a diagonal of $S_{n+1}$. If the length of a side of $S_1$ is 10 *cm*, then for which of the following values of *n* is the area of $S_n$ less than 1 sq. *cm*.

(a) 7 (b) 8 (c) 9 (d) 10

## *Problems concerning to geometrical conditions*

### *Basic Level*

23. The three points (– 2, 2), (8, – 2) and (– 4, – 3) are the vertices of **[Rajasthan PET 1987]**

(a) An isosceles triangle (b) An equilateral triangle (c) A right angled triangle (d) None of these

24. The points $A(-4, -1); B(-2, -4); C(4, 0)$ and $D(2, 3)$ are the vertices of a

(a) Parallelogram (b) Rectangle (c) Rhombus (d) None of these

25. Two opposite vertices of a rectangle are (1,3) and (5,1). If the other two vertices of the rectangle lie on the line $y - x + \lambda = 0$, then $\lambda =$

(a) 1 (b) – 1 (c) 2 (d) None of these

26. Three vertices of a parallelogram are (1, 3) (2, 0) and (5, 1). Then its fourth vertex is **[Rajasthan PET 1988, 2001]**

(a) (3, 3) (b) (4, 4) (c) (4, 0) (d) (0, – 4)

27. The quadrilateral formed by the vertices (– 1, 1), (0, – 3), (5, 2) and (4, 6) will be **[Rajasthan PET 1986]**

(a) Square (b) Parallelogram (c) Rectangle (d) Rhombus

28. The triangle formed by the lines $x + y = 0$, $3x + y - 4 = 0$ and $x + 3y = 4$ is **[IIT 1983; MNR 1992; Rajasthan PET 1995; UPSEAT 2001]**

(a) Equilateral (b) Isosceles (c) Right angled (d) None of these

29. The following points $A(2a, 4a), B(2a, 6a)$ and $C(2a + \sqrt{3}a, 5a)$, $(a > 0)$ are the vertices of

(a) An acute angled triangle (b) An right angled triangle (c) An isosceles triangle (d)

30. The triangle joining the points $P(2,7), Q(4,-1), R(-2,6)$ is **[MP PET 1997]**

(a) Equilateral triangle (b) Right-angled triangle (c) Isosceles triangle (d) Scalene triangle

31. The points (1, 3) and (5, 1) are the opposite vertices of a rectangle. The other two vertices lie on the line $y = 2x + c$, then the value of *c* will be **[IIT 1981]**

(a) 4 (b) – 4 (c) 2 (d) – 2

32. If the three vertices of a rectangle taken in order are the points (2, –2), (8, 4) and (5, 7). The coordinates of fourth vertex are

[Kurukshetra CEE 1993]

(a) (1, 1) (b) (1, –1) (c) (–1, 1) (d) None of these

33. If vertices of a quadrilateral are $A(0,0), B(3,4), C(7,7)$ and $D(4,3)$ then quadrilateral *ABCD* is a [Rajasthan PET 1986]

(a) Parallelogram (b) Rectangle (c) Square (d) Rhombus

34. The coordinates of the third vertex of an equilateral triangle whose two vertices are at (3, 4) and (–2, 3) are

(a) (1, 1) or (1, –1) (b) $\left(\frac{1+\sqrt{3}}{2}, \frac{7-5\sqrt{3}}{2}\right)$ or $\left(\frac{1-\sqrt{3}}{2}, \frac{7+5\sqrt{3}}{2}\right)$

(c) $(-\sqrt{3}, \sqrt{3})$ or $(\sqrt{3}, -\sqrt{3})$ (d) None of these

35. The quadrilateral joining the points (1, –2); (3, 0); (1, 2) and (–1, 0) is [Rajasthan PET 1999]

(a) Parallelogram (b) Rectangle (c) Square (d) Rhombus

36. If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$, then the two triangle with vertices $(x_1, y_1); (x_2, y_2); (x_3, y_3)$ and $(a_1, b_1); (a_2, b_2); (a_3, b_3)$ must be

[IIT 1985]

(a) Similar (b) Congruent (c) Never congruent (d) None of these

37. All points lying inside the triangle formed by the points (1, 3), (5, 0) and (–1, 2) satisfy [IIT 1986; Kurukshetra CEE 1998]

(a) $3x + 2y \geq 0$ (b) $2x + y - 13 \leq 0$ (c) $2x - 3y - 12 \leq 0$ (d) All of these

38. The common property of points lying on *x*-axis, is [MP PET 1988]

(a) $x = 0$ (b) $y = 0$ (c) $a = 0, y = 0$ (d) $y = 0, b = 0$

39. Vertices of a figure are (– 2, 2); (– 2, – 1); (3, –1); (3, 2), it is a [Karnataka CET 1998]

(a) Square (b) Rhombus (c) Rectangle (d) Parallelogram

40. If *ABCD* is a quadrilateral, if the mid point of consecutive sides *AB, BC, CD* and *DA* are combined by straight lines, then the quadrilateral *PQRS* is always [Orissa JEE 2002]

(a) Square (b) Parallelogram (c) Rectangle (d) Rhombus

41. Three vertices of a parallelogram taken in order are $(-1, -6), (2, -5)$ and $(7, 2)$. The fourth vertex is

(a) (1, 4) (b) (4, 1) (c) (1, 1) (d) (4, 4)

42. If $P(1, 2), Q(4, 6) R(5, 7)$ and $S(a, b)$ are the vertices of a parallelogram *PQRS*, then [IIT 1998]

(a) $a = 2, b = 4$ (b) $a = 3, b = 4$ (c) $a = 2, b = 3$ (d) $a = 3, b = 5$

## *Advance Level*

43. The sides of a triangle are $3x + 4y, 4x + 3y$ and $5x + 5y$ where $x, y > 0$, then the triangle is [AIEEE 2002]

(a) Right angled (b) Obtuse angled (c) Equilateral (d) None of these

44. If the vertices of triangle have integral coordinates then the triangle is [IIT 1975; MP PET 1983]

(a) Equilateral (b) Never equilateral (c) Isosceles (d) None of these

45. The opposite angular points of a square are (3, 4) and (1, –1). Then the coordinates of other two vertices are [Roorkee 1985]

(a) $D\left(\frac{1}{2},\frac{9}{2}\right); B\left(-\frac{1}{2},\frac{5}{2}\right)$ (b) $D\left(-\frac{1}{2},\frac{9}{2}\right); B\left(\frac{1}{2},\frac{5}{2}\right)$ (c) $D\left(\frac{9}{2},\frac{1}{2}\right); B\left(-\frac{1}{2},\frac{5}{2}\right)$ (d) None of these

46. The quadrilateral formed by the lines $ax \pm by \pm c = 0$ is [Rajasthan PET 1998]

(a) Square (b) Rectangle (c) Rhombus (d) Parallelogram

## Section Formulae

### Basic Level

47. Point $\left(\frac{1}{2},\frac{-13}{4}\right)$ divides the line joining the points (3, – 5) and (– 7, 2) in the ratio of

(a) 1 : 3 internally (b) 3 : 1 internally (c) 1 : 3 externally (d) 3 : 1 externally

48. In what ratio does the *y*-axis divide the join of (–3, –4) and (1, –2) [Rajasthan PET 1995]

(a) 1 : 3 (b) 2 : 3 (c) 3 : 1 (d) None of these

49. The points which trisect the line segment joining the points (0, 0) and (9, 12) are [Rajasthan PET 1986]

(a) (3, 4), (6, 8) (b) (4, 3), (6, 8) (c) (4, 3), (8, 6) (d) (3, 4), (8, 6)

50. If the point dividing internally the line segment joining the points $(a,b)$ and $(5,7)$ in the ratio 2 : 1 be (4, 6) then

(a) $a=1, b=2$ (b) $a=2, b=-4$ (c) $a=2, b=4$ (d) $a=-2, b=4$

51. If *A* and *B* are the points $(-3,4)$ and (2, 1). Then the co-ordinates of point *C* on *AB* produced such that *AC* = 2*BC* are

(a) (2, 4) (b) (3, 7) (c) (7, –2) (d) $\left(-\frac{1}{2},\frac{5}{2}\right)$

52. The line segment joining the points (1, 2) and (– 2, 1) is divided by the line $3x+4y=7$ in the ratio

(a) 3 : 4 (b) 4 : 3 (c) 9 : 4 (d) 4 : 9

### Advance Level

53. If the points $P_1, P_2, P_3.....$ are the middle points of line segments $AB, P_1B, P_2B,.....$ respectively and particles of masses $m; \frac{m}{2}, \frac{m}{2^2},.....$ are placed respectively on these points. If *G* is the mass-centre of so placed infinite particles and $\overline{BG} = p\,\overline{BA}$, then *p* is [MP PET 1998]

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

54. If coordinates of the points *A* and *B* are (2, 4) and (4, 2) respectively and point *M* is such that *A*-*M*-*B* also *AB* = 3*AM*, then the coordinates of *M* are

(a) $\left(\frac{8}{3},\frac{10}{3}\right)$ (b) $\left(\frac{10}{3},\frac{14}{4}\right)$ (c) $\left(\frac{10}{3},\frac{6}{3}\right)$ (d) $\left(\frac{13}{4},\frac{10}{4}\right)$

55. The mid-points of sides of a triangle are (2, 1), (–1, –3) and (4, 5). Then the coordinates of its vertices are

(a) (7, 9), (– 3, – 7), (1, 1) (b) (– 3, – 7), (1, 1), (2, 3) (c) (1, 1), (2, 3), (– 5, 8) (d) None of these

56. The coordinates of the points *A, B, C* are $(x_1, y_1)$, $(x_2, y_2), (x_3, y_3)$ and *D* divides the line *AB* in the ratio $l : k$. If *P* divides the line *DC* in the ratio $m : k + l$, then the coordinates of *P* are

(a) $\left(\frac{kx_1 + lx_2 + mx_3}{k + l + m}, \frac{ky_1 + ly_2 + my_3}{k + l + m}\right)$ (b) $\left(\frac{lx_1 + mx_2 + kx_3}{l + m + k}, \frac{ly_1 + my_2 + ky_3}{l + m + k}\right)$

(c) $\left(\frac{mx_1 + kx_2 + lx_3}{m + k + l}, \frac{my_1 + ky_2 + ly_3}{m + k + l}\right)$ (d) None of these

## *Some points related to Triangle*

### *Basic Level*

57. If the coordinates of the vertices of a triangle be $(1, a), (2, b)$ and $(c^2, 3)$, then the centroid of the triangle

(a) Lies at the origin (b) Cannot lie on *x*-axis (c) Cannot lie on *y*-axis (d) None of these

58. If $A(4, -3), B(3, -2)$ and $C(2, 8)$ are the vertices of a triangle, then its centroid will be [Rajasthan PET 1984, 1986]

(a) (– 3, 3) (b) (3, 3) (c) (3, 1) (d) (1, 3)

59. Two vertices of a triangle are (5, 4) and (– 2, 4). If its centroid is (5, 6) then the third vertex has the coordinates [MP PET 1993]

(a) (12, 10) (b) (10, 12) (c) (–10, 12) (d) (12, –10)

60. The centroid of a triangle, whose vertices are (2, 1), (5, 2) and (3, 4) is [IIT 1964]

(a) $\left(\frac{8}{3},\frac{7}{3}\right)$ (b) $\left(\frac{10}{3},\frac{7}{3}\right)$ (c) $\left(-\frac{10}{3},\frac{7}{3}\right)$ (d) $\left(\frac{10}{3},-\frac{7}{3}\right)$

61. If the middle points of the sides of a triangle be (–2, 3), (4, –3) and (4, 5), then the centroid of the triangle is

(a) (5/3, 2) (b) (5/6, 1) (c) (2, 5/3) (d) (1, 5/6)

62. If $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle, then the excentre with respect to *B* is [Rajasthan PET 2000]

(a) $\left(\frac{ax_1 - bx_2 + cx_3}{a - b + c}, \frac{ay_1 - by_2 + cy_3}{a - b + c}\right)$ (b) $\left(\frac{ax_1 + bx_2 - cx_3}{a + b - c}, \frac{ay_1 + by_2 - cy_3}{a + b - c}\right)$

(c) $\left(\frac{ax_1 - bx_2 - cx_3}{a - b - c}, \frac{ay_1 - by_2 - cy_3}{a - b - c}\right)$ (d) None of these

63. If two vertices of an equilateral triangle have integral co-ordinates then the third vertex will have

(a) Integral co-ordinates (b) Co-ordinates which are rational

(c) At least one co-ordinate irrational (d) Co-ordinates which are irrational

64. If the orthocentre and centroid of triangle are (–3, 5), (3, 3), then the circumcentre is [Kurukshetra CEE 1999]

(a) (6, 2) (b) (0, 8) (c) (6, –2) (d) (0, 4)

65. The centroid and a vertex of an equilateral triangle are (1, 1) and (1, 2) respectively. Another vertex of the triangle can be

(a) $\left(\frac{2-\sqrt{3}}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{2+3\sqrt{3}}{2}, \frac{1}{2}\right)$ (c) $\left(\frac{2+\sqrt{3}}{2}, \frac{1}{2}\right)$ (d) None of these

66. The incentre of triangle formed by lines $x=0$, $y=0$ and $3x+4y=12$ is [Rajasthan PET 1990]

(a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) (1, 1) (c) $\left(1, \frac{1}{2}\right)$ (d) $\left(\frac{11}{2}, 1\right)$

67. Orthocentre of triangle with vertices (0, 0), (3, 4), (4, 0) is [IIT Screening 2003]

(a) $\left(3, \frac{5}{4}\right)$ (b) (3, 12) (c) $\left(3, \frac{3}{4}\right)$ (d) (3, 9)

68. Orthocentre of the triangle whose vertices are (0, 0)), (2, – 1) and (1, 3) is [ISM Dhanbad 1970; IIT 1967, 1974]

(a) $\left(\frac{4}{7}, \frac{1}{7}\right)$ (b) $\left(-\frac{4}{7}, -\frac{1}{7}\right)$ (c) (– 4, – 1) (d) (4, 1)

69. The orthocentre of the triangle formed by the lines $4x-7y+10=0$, $x+y=5$ and $7x+4y=15$ is [IIT 1969, 1976]

(a) (1, 2) (b) (1, –2) (c) (–1, – 2) (d) (–1, 2)

70. Coordinates of the orthocentre of the triangle whose sides are $x=3$, $y=4$ and $3x+4y=6$, will be [MNR 1989]

(a) (0, 0) (b) (3, 0) (c) (0, 4) (d) (3, 4)

71. The orthocentre of the triangle formed by (0, 0), (8, 0), (4, 6) is [EAMCET 1991]

(a) $\left(4, \frac{8}{3}\right)$ (b) (3, 4) (c) (4, 3) (d) (–3, 4)

72. If the line $3x+4y-24=0$ cuts the *x*-axis in *A* and *y*-axis in *B*, then incentre of $\Delta OAB$ (where *O* is the origin) is

(a) (1, 2) (b) (2, 2) (c) (12, 12) (d) (2, 12)

73. The distance between the orthocentre and circumcentre of the triangle with vertices (0, 0) , (0, *a*) and (*b*, 0) is

(a) $\frac{\sqrt{|a^2-b^2|}}{2}$ (b) $a+b$ (c) $a-b$ (d) $\frac{\sqrt{a^2+b^2}}{2}$

74. The incentre of the triangle formed by (0, 0); (5, 12); (16, 12) is [EAMCET 1984]

(a) (9, 7) (b) (7, 9) (c) (–9, 7) (d) (–7, 9)

75. If two vertices of a triangles are (6, 4); (2, 6) and its centroid is (4, 6), then the third vertex is [Rajasthan PET 1996]

(a) (4, 8) (b) (8, 4) (c) (6, 4) (d) None of these

76. If the vertices of a triangle be (*a*, 1); (*b*, 3) and (4, *c*), then the centroid of the triangle will lie on *x*-axis if

(a) $a+c=-4$ (b) $a+b=-4$ (c) $c=-4$ (d) $b+c=-4$

77. The vertices of a triangle are (0, 0), (3, 0) and (0, 4). Its orthocentre is at [MNR 1982; Rajasthan PET 1997; DCE 1994]

(a) (0, 0) (b) $\left(1, \frac{4}{3}\right)$ (c) $\left(\frac{3}{2}, 2\right)$ (d) None of these

## Advance Level

78. The equations of the sides of a triangle are $x+y-5=0; x-y+1=0$ and $y-1=0$, then the coordinates of the circumcentre are [MP PET 1996]

(a) (2, 1) (b) (1, 2) (c) (2, –2) (d) (1, – 2)

79. The mid points of the sides of a triangle are (5, 0); (5, 12) and (0, 12). The orthocentre of this triangle is

(a) (0, 0) (b) (10, 0) (c) (0, 24) (d) $\left(\frac{13}{3}, 8\right)$

80. The orthocentre of the triangle with vertices $\left(2, \frac{\sqrt{3}-1}{2}\right); \left(\frac{1}{2}, -\frac{1}{2}\right)$ and $\left(2, -\frac{1}{2}\right)$ is [IIT 1993]

(a) $\left(\frac{3}{2}, \frac{\sqrt{3}-3}{6}\right)$ (b) $\left(2, -\frac{1}{2}\right)$ (c) $\left(\frac{5}{4}, \frac{\sqrt{3}-2}{5}\right)$ (d) $\left(\frac{1}{2}, -\frac{1}{2}\right)$

81. If the coordinates of the vertices of a triangle are rational numbers then which of the following points of the triangle will always have rational coordinates

(a) Centroid (b) Incentre (c) Circumcentre (d) Orthocentre

82. In the $\Delta ABC$, the coordinates of $B$ are (0, 0), $AB = 2, \angle ABC = \frac{\pi}{3}$ and the middle point of $BC$ has the coordinates (2, 0). The centroid of the triangle is

(a) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{5}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ (c) $\left(\frac{4+\sqrt{3}}{3}, \frac{1}{3}\right)$ (d) None of these

83. The vertices of triangle are (6, 0), (0, 6) and (6, 6). The distance between its circumcentre and centroid is

(a) $2\sqrt{2}$ (b) 2 (c) $\sqrt{2}$ (d) 1

84. Two vertices of a triangle are (5, –1) and (–2, 3). If orthocentre is the origin then co-ordinates of the third vertex are

(a) (7, 4) (b) (–4, 7) (c) (4, –7) (d) (– 4, – 7)

85. The orthocentre of the triangle formed by the lines $x+y=1$, $2x+3y=6$ and $4x-y+4=0$ lies in quadrant [IIT 1985]

(a) First (b) Second (c) Third (d) Fourth

86. Two vertices of a triangle are (4, –3) and (–2, 5). If the orthocentre of the triangle is at (1, 2), then the third vertex is [Roorkee 1987]

(a) (– 33, –26) (b) (33, 26) (c) (26, 33) (d) None of these

87. The equations to the sides of a triangle are $x-3y=0$, $4x+3y=5$ and $3x+y=0$. The line $3x-4y=0$ passes through [EAMCET 1994]

(a) The incentre (b) The centroid (c) The circumcentre (d) The orthocentre of the triangle

88. The vertices of a triangle are $|at_1t_2; a(t_1+t_2)|$, $|at_2t_3, a(t_2+t_3)|$, $|at_3t_1, a(t_3+t_1)|$, then the coordinates of its orthocentre are [IIT 1983]

(a) $|a, a(t_1+t_2+t_3+t_1t_2t_3)|$ (b) $[-a, a(t_1+t_2+t_3+t_1t_2t_3)]$

(c) $[-a, (t_1+t_2+t_3+t_1t_2t_3), a]$ (d) None of these

89. The equations of the three sides of a triangle are $x=2, y+1=0$ and $x+2y=4$. The coordinates of the circumcentre of the triangle are

(a) (4, 0) (b) (2, –1) (c) (0, 4) (d) None of these

## *Area of Some geometrical figures*

### *Basic Level*

90. The area of the triangle with vertices at (–4, 1), (1, 2), (4, – 3) is [EAMCET 1980]

(a) 14 (b) 16 (c) 15 (d) None of these

91. If the coordinates of the points *A, B, C* be (4, 4) (3, –2) and (3, – 16) respectively, then the area of the triangle *ABC* is [MP PET 1982]

(a) 27 (b) 15 (c) 18 (d) 7

92. If the vertices of a triangle are (5, 2), (2/3, 2) and (–4, 3), then the area of the triangle is [Kurukshetra CEE 2002]

(a) $\frac{28}{6}$ (b) $\frac{5}{2}$ (c) 43 (d) $\frac{13}{6}$

93. The area of a triangle whose vertices are (1, –1), (–1, 1) and (–1, –1) is given by [AMU 1981; Rajasthan PET 1989; MP PET 1993]

(a) 2 (b) $\frac{1}{2}$ (c) 1 (d) 3

94. The vertices of a triangle *ABC* are $(\lambda, 2-2\lambda)$, $(-\lambda+1, 2\lambda)$ and $(-4-\lambda, 6-2\lambda)$. If its area be 70 units then number of integral values of $\lambda$ is

(a) 1 (b) 2 (c) 4 (d) 0

95. The area of the pentagon whose vertices are (1, 2), (–3, 2), (4, 5), (–3, 3) and (–3, 0) is

(a) 15/2 unit$^2$ (b) 30 unit$^2$ (c) 45 unit$^2$ (d) None of these

### *Advance Level*

96. If $A(6,3), B(-3,5), C(4,-2)$ and $D(x,3x)$ are four points. If the ratio of area of $\Delta DBC$ and $\Delta ABC$ is 1 : 2, then the value of *x* will be [IIT 1959]

(a) $\frac{11}{8}$ (b) $\frac{8}{11}$ (c) 3 (d) None of these

97. The point A divides the join of the points (– 5, 1) and (3, 5) in the ratio *k* : 1 and the coordinates of the points *B* and *C* are (1, 5) and (7, – 2) respectively. If the area of the triangle *ABC* be 2 units, then *k* = [IIT 1967; Kurukshetra CEE 1998]

(a) 6, 7 (b) 31/9, 9 (c) 7, 31/9 (d) 7, 9

98. The area of a triangle is 5. If two of its vertices are (2, 1), (3, –2) and the third vertex lies on the line $y = x + 3$, then the third vertex is

[IIT 1978; UPSEAT 1999]

(a) $\left(-\frac{7}{2}, -\frac{13}{2}\right)$ (b) $\left(-\frac{7}{2}, \frac{13}{2}\right)$ (c) $\left(\frac{7}{2}, -\frac{13}{2}\right)$ (d) $\left(\frac{7}{2}, \frac{13}{2}\right)$

99. The area of the triangle formed by the lines $7x-2y+10=0, 7x+2y-10=0$ and $y+2=0$ is [IIT 1977]

(a) 8 sq. units (b) 12 sq. units (c) 14 sq. units (d) None of these

100. Area of the triangle with vertices (*a*, *b*), $(x_1, y_1)$ and $(x_2, y_2)$ where $a, x_1, x_2$ are in G.P. with common ratio '*r*' and $b, y_1, y_2$ are in G.P. with common ratio '*s*' is

(a) $ab(r-1)(s-1)(s-r)$ (b) $\frac{1}{2}ab(r+1)(s+1)(s-r)$ (c) $\frac{1}{2}ab(r-1)(s-1)(s-r)$ (d) $ab(r+1)(s+1)(r-s)$

101. If the area of the triangle whose vertices are $(b,c),(c,a)$ and $(a,b)$ is $\Delta$, then the area of triangle whose vertices are $(ac-b^2, ab-c^2)$ $(ba-c^2, bc-a^2)$ and $(cb-a^2, ca-b^2)$ is

(a) $\Delta^2$ (b) $(a+b+c)^2\Delta$ (c) $a\Delta + b\Delta^2$ (d) None of these

102. $P(2,1), Q(4,-1), R(3,2)$ are the vertices of a triangle and if through *P* and *R* lines parallel to opposite sides are drawn to intersect in *S*, then the area of *PQRS* is

(a) 6 (b) 4 (c) 8 (d) 12

103. An equilateral triangle has each side equal to *a*. If the coordinates of its vertices are $(x_1,y_1);(x_2,y_2);(x_3,y_3)$, then the square of the determinant $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ equals

(a) $3a^4$ (b) $\frac{3a^4}{4}$ (c) $4a^4$ (d) None of these

104. Area of a $\Delta ABC = 20$ units and its vertices *A* and *B* are (–5, 0) and (3, 0) respectively. If its vertex *C* lies on the line $x - y = 2$, then *C* is [IIT 1990]

(a) (3, 5) (b) (– 3, – 5) (c) (– 5, 7) (d) None of these

105. Point *P* divides the line segment joining $A(-5,1)$ and $B(3,5)$ internally in the ratio $\lambda : 1$. If $Q \equiv (1,5), R \equiv (7,2)$ and area of $\Delta PQR = 2,$ then $\lambda$ equals [Kurukshetra CEE 1998]

(a) 23 (b) 31/9 (c) 29/5 (d) None of these

## *Collinearity*

### *Basic Level*

106. Three points $(p+1,1),(2p+1,3)$ and $(2p+2,2p)$ are collinear if $p =$ [MP PET 1986]

(a) – 1 (b) 1 (c) 2 (d) 0

107. If the points (*a*, 0), (0, *b*) and (1, 1) are collinear, then

(a) $\frac{1}{a^2}+\frac{1}{b^2}=1$ (b) $\frac{1}{a^2}-\frac{1}{b^2}=1$ (c) $\frac{1}{a}+\frac{1}{b}=1$ (d) $\frac{1}{a}-\frac{1}{b}=1$

108. If the points (*a*, *b*), $(a', b')$ and $(a-a', b-b')$ are collinear, then [Rajasthan PET 1999]

(a) $ab' = a'b$ (b) $ab = a'b'$ (c) $aa' = bb'$ (d) $a^2 + b^2 = 1$

109. If the points $(k, 2-2k), (1-k, 2k)$ and $(-k-4, 6-2k)$ be collinear, then the possible values of *k* are [AMU 1978; Rajasthan PET 1997]

(a) $\frac{1}{2}, -1$ (b) $1, -\frac{1}{2}$ (c) $1, -2$ (d) $2, -1$

110. If the points (–5, 1), (*p*, 5) and (10, 7) are collinear, then the value of *p* will be [MP PET 1984]

(a) 5 (b) 3 (c) 4 (d) 7

111. If the points $(-2, -5), (2, -2), (8, a)$ are collinear, then the value of *a* is [MP PET 2002]

(a) $-\frac{5}{2}$ (b) $\frac{5}{2}$ (c) $\frac{3}{2}$ (d) $\frac{1}{2}$

112. If the points (5, 5), (10, *K*) and (–5, 1) are collinear, then *K* = [MP PET 1994, 1999; Rajasthan PET 2003]

(a) 3 (b) 5 (c) 7 (d) 9

113. The points $(-a, -b), (a, b), (a^2, ab)$ are

(a) Vertices of an equilateral triangle (b) Vertices of a right angled triangle

(c) Vertices of an isosceles triangle (d) Collinear

114. The points $(3a, 0), (0, 3b)$ and $(a, 2b)$ are [MP PET 1982]

(a) Vertices of an equilateral triangle (b) Vertices of an isosceles triangle

(c) Vertices of a right angled isosceles triangle (d) Collinear

115. The points (*a*, *b*), (*c*, *d*) and $\left(\frac{kc+la}{k+l}, \frac{kd+lb}{k+l}\right)$ are

(a) Vertices of an equilateral triangle (b) Vertices of an isosceles triangle

(c) Vertices of a right angled triangle (d) Collinear

## *Advance Level*

116. *A*, *B*, *C* are the points (*a*, *p*), (*b*, *q*) and (*c*, *r*) respectively such that *a*, *b*, *c* are in A.P. and *p*, *q*, *r* in G.P. If the points are collinear, then

(a) $p = q = r$ (b) $p^2 = q$ (c) $q^2 = r$ (d) $r^2 = p$

117. *A*, *B*, *C* are three collinear points such that *AB* = 2.5 and the co-ordinates of *A* and *C* are respectively (3, 4) and (11, 10), then the co-ordinates of the point *B* are

(a) $\left(5, \frac{11}{2}\right)$ (b) $\left(5, \frac{5}{2}\right)$ (c) $\left(1, \frac{11}{2}\right)$ (d) $\left(1, \frac{5}{2}\right)$

118. The points $(x, 2x), (2y, y)$ and $(3, 3)$ are collinear

(a) For all values of $(x, y)$ (b) 2 is A.M. of *x*, *y* (c) 2 is G.M. of *x*, *y* (d) 2 is H.M. of *x*, *y*

119. If $t_1, t_2$ and $t_3$ are distinct, the points $(t_1, 2at_1 + at_1^3), (t_2, 2at_2 + at_2^3)$ and $(t_3, 2at_3 + at_3^3)$ are collinear if

(a) $t_1t_2t_3 = -1$ (b) $t_1 + t_2 + t_3 = t_1t_2t_3$ (c) $t_1 + t_2 + t_3 = 0$ (d) $t_1 + t_2 + t_3 = -1$

120. The points $(-a, -b), (0,0), (a,b)$ and $(a^2, ab)$ are [IIT 1979; Kurukshetra CEE 1993; Jamia Millia Entrance Exam. 2001]

(a) Collinear (b) Vertices of a rectangle (c) Vertices of a parallelogram (d) None of these

## Transformation of Axes

### Basic Level

121. The new coordinates of a point (4, 5), when the origin is shifted to the point (1, –2) are [MNR 1988; IIT 1989; UPSEAT 2000]

(a) (5, 3) (b) (3, 5) (c) (3, 7) (d) None of these

122. The co-ordinate axes are rotated through an angle $135°$. If the co-ordinates of a point *P* in the new system are known to be $(4, -3)$, then the co-ordinates of *P* in the original system are [EAMCET 2003]

(a) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (b) $\left(\frac{1}{\sqrt{2}}, \frac{-7}{\sqrt{2}}\right)$ (c) $\left(\frac{-1}{\sqrt{2}}, \frac{-7}{\sqrt{2}}\right)$ (d) $\left(\frac{-1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$

123. If the axes be rotated through an angle of $60°$ in the clockwise direction, the point (4, 2) in the new system was formally

(a) $(2-\sqrt{3}, 2\sqrt{3}+1)$ (b) $(2+\sqrt{3}, -2\sqrt{3}+1)$ (c) $(2-\sqrt{3}, 1-2\sqrt{3})$ (d) None of these

### Advance Level

124. Without changing the direction of coordinate axes origin is transferred to (*h*, *k*), so that the linear (one degree) terms in the equation $x^2 + y^2 - 4x + 6y - 7 = 0$ are eliminated. Then the point (*h*, *k*) is

(a) (3, 2) (b) (– 3, 2) (c) (2, – 3) (d) None of these

125. The point (4, 1) undergoes the following two successive transformations

(i) reflection about the line $y = x$

(ii) rotation through a distance 2 units along the positive *x*-axis

Then the final coordinates of the point are

(a) (4, 3) (b) (3, 4) (c) (1, 4) (d) (7/2, 7/2)

## Locus

### Basic Level

126. Two points $A$ and $B$ have coordinates (1, 0) and (–1, 0) respectively and $Q$ is a point which satisfies the relation $AQ - BQ = \pm 1$. The locus of $Q$ is [MP PET 1986]

(a) $12x^2 + 4y^2 = 3$ (b) $12x^2 - 4y^2 = 3$ (c) $12x^2 - 4y^2 + 3 = 0$ (d) $12x^2 + 4y^2 + 3 = 0$

127. A point moves such that the sum of its distances from two fixed points ($ae$, 0) and ($-ae$, 0) is always $2a$. Then equation of its locus is

[MNR 1981]

(a) $\frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1$ (b) $\frac{x^2}{a^2} - \frac{y^2}{a^2(1-e^2)} = 1$ (c) $\frac{x^2}{a^2(1-e^2)} + \frac{y^2}{a^2} = 1$ (d) None of these

128. The locus of a point whose distance from the point $(-g, -f)$ is always '$a$', will be (where $k = g^2 + f^2 - a^2$)

(a) $x^2 + y^2 + 2gx + 2fy + k = 0$ (b) $x^2 - y^2 + 2gx + 2fy + k = 0$

(c) $x^2 + y^2 + 2xy + 2gx + 2fy + k = 0$ (d) None of these

129. The coordinates of the points A and B are $(a, 0)$ and $(-a, 0)$ respectively. If a point $P$ moves so that $PA^2 - PB^2 = 2k^2$, when $k$ is a constant, then the equation to the locus of the point $P$ is

(a) $2ax - k^2 = 0$ (b) $2ax + k^2 = 0$ (c) $2ay - k^2 = 0$ (d) $2ay + k^2 = 0$

130. If the distance of any point $P$ from the points $A(a+b, a-b)$ and $B(a-b, a+b)$ are equal, then the locus of $P$ is

[Karnataka CET 2003]

(a) $x - y = 0$ (b) $ax + by = 0$ (c) $bx - ay = 0$ (d) $x + y = 0$

131. The locus of a point whose difference of distance from points (3, 0) and (–3, 0) is 4, is [MP PET 2002]

(a) $\frac{x^2}{4} - \frac{y^2}{5} = 1$ (b) $\frac{x^2}{5} - \frac{y^2}{4} = 1$ (c) $\frac{x^2}{2} - \frac{y^2}{3} = 1$ (d) $\frac{x^2}{3} - \frac{y^2}{2} = 1$

132. If $A$ and $B$ are two fixed points in a plane and $PA - PB$ = constant, then the locus of $P$ is

(a) Hyperbola (b) Circle (c) Parabola (d) Ellipse

133. If $A$ and $B$ are two points in a plane, so that $PA + PB$ = constant, then the locus of $P$ is [MNR 1991]

(a) Hyperbola (b) Circle (c) Parabola (d) Ellipse

134. The equation of the locus of all points equidistant from the point (4, 2) and the $x$-axis, is [Kurukshetra CEE 1993]

(a) $x^2 + 8x + 4y - 20 = 0$ (b) $x^2 - 8x - 4y + 20 = 0$ (c) $y^2 - 4y - 8x + 20 = 0$ (d) None of these

135. The locus of a point which moves so that it is always equidistant from the points $A(a, 0)$ and $B(-a, 0)$ is

(a) A circle (b) Perpendicular bisector of the line segment $AB$

(c) A line parallel to $x$-axis (d) None of these

136. The locus of a point which moves so that its distance from $x$-axis is double of its distance from $y$-axis is [AMU 1978; MP PET 1984]

(a) $x = 2y$ (b) $y = 2x$ (c) $x = 5y + 1$ (d) $y = 2x + 3$

137. $O$ is the origin and $A$ is the point (3, 4). If a point $P$ moves so that the line segment $OP$ is always parallel to the line segment $OA$, then the equation to the locus of $P$ is

(a) $4x - 3y = 0$ (b) $4x + 3y = 0$ (c) $3x + 4y = 0$ (d) $3x - 4y = 0$

138. If A and B are two fixed points in a plane and *P* is another variable point such that $PA^2 + PB^2 =$ constant, then the locus of the point *P* is

(a) Hyperbola (b) Circle (c) Parabola (d) Ellipse

139. If sum of distances of a point from the origin and line $x = 2$ is 4, then its locus is [Rajasthan PET 1997]

(a) $x^2 - 12y = 36$ (b) $y^2 + 12x = 36$ (c) $y^2 - 12x = 36$ (d) $x^2 + 12y = 36$

140. The coordinates of the points *A* and *B* are $(ak, 0)$ and $\left(\frac{a}{k}, 0\right)$, $(k = \pm 1)$. If a point *P* moves so that $PA = k\,PB$, then the equation to the locus of *P* is

(a) $k^2(x^2 + y^2) - a^2 = 0$ (b) $x^2 + y^2 - k^2a^2 = 0$ (c) $x^2 + y^2 + a^2 = 0$ (d) $x^2 + y^2 - a^2 = 0$

141. The equation of the locus of a point whose distance from (*a*, 0) is equal to its distance from *y*-axis, is

(a) $y^2 - 2ax = a^2$ (b) $y^2 - 2ax + a^2 = 0$ (c) $y^2 + 2ax + a^2 = 0$ (d) $y^2 + 2ax = a^2$

142. The locus of the point of intersection of lines $x\cos\alpha + y\sin\alpha = a$ and $x\sin\alpha - y\cos\alpha = b$ is ($\alpha$ is a variable)

(a) $2(x^2 + y^2) = a^2 + b^2$ (b) $x^2 - y^2 = a^2 - b^2$ (c) $x^2 + y^2 = a^2 + b^2$ (d) None of these

143. Two points A and B move on the *x*- axis and the *y*-axis respectively such that the distance between the two points is always the same. The locus of the middle point of *AB* is

(a) A straight line (b) A circle (c) A parabola (d) An ellipse

## Advance Level

144. The locus of *P* such that area of $\Delta PAB = 12\,sq.$ units, where $A(2, 3)$ and $B(-4, 5)$ is [EAMCET 1989]

(a) $(x + 3y - 1)\ (x + 3y - 23) = 0$ (b) $(x + 3y + 1)(x + 3y - 23) = 0$

(c) $(3x + y - 1)(3x + y - 23) = 0$ (d) $(3x + y + 1)(3x + y + 23) = 0$

145. Locus of centroid of the triangle whose vertices are $(a\cos t, a\sin t), (b\sin t, -b\cos t)$ and (1, 0), where *t* is a parameter is [AIEEE 2003]

(a) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$ (b) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

(c) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$ (d) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$

146. If *A* is (2, 5), *B* is (4, –11) and *C* lies on $9x + 7y + 4 = 0$, then the locus of the centroid of the $\Delta ABC$ is a straight line parallel to the straight line [MP PET 1986]

(a) $7x - 9y + 4 = 0$ (b) $9x - 7y - 4 = 0$ (c) $9x + 7y + 4 = 0$ (d) $7x + 9y + 4 = 0$

147. Two fixed points are $A(a, 0)$ and $B(-a, 0)$. If $\angle A - \angle B = \theta$, then the locus of point *C* of triangle *ABC* will be [Roorkee 1982]

(a) $x^2 + y^2 + 2xy\tan\theta = a^2$ (b) $x^2 - y^2 + 2xy\tan\theta = a^2$ (c) $x^2 + y^2 + 2xy\cot\theta = a^2$ (d) $x^2 - y^2 + 2xy\cot\theta = a^2$

148. If $A(-a, 0)$ and $B(a, 0)$ are two fixed points, then the locus of the point on which the line *AB* subtends the right angle, is

(a) $x^2+y^2=2a^2$ (b) $x^2-y^2=a^2$ (c) $x^2+y^2+a^2=0$ (d) $x^2+y^2=a^2$

149. The coordinates of the points *O*, *A* and *B* are (0, 0), (0, 4) and (6, 0) respectively. If a point *P* moves such that the area of $\Delta POA$ is always twice the area of $\Delta POB$, then the equation to both parts of the locus of *P* is [IIT 1964]

(a) $(x-3y)(x+3y)=0$ (b) $(x-3y)(x+y)=0$ (c) $(3x-y)(3x+y)=0$ (d) None of these

150. A stick of length *l* rests against the floor and a wall of a room. If the stick begins to slide on the floor, then the locus of its middle point is

(a) A straight line (b) Circle (c) Parabola (d) Ellipse

151. Given the points $A(0,4)$ and $B(0,-4)$. Then the equation of the locus of the point $P(x,y)$ such that $|AP-BP|=6$, is

[IIT 1983; MP PET 1994]

(a) $\frac{x^2}{7}+\frac{y^2}{9}=1$ (b) $\frac{x^2}{9}+\frac{y^2}{7}=1$ (c) $\frac{x^2}{7}-\frac{y^2}{9}=1$ (d) $\frac{y^2}{9}-\frac{x^2}{7}=1$

152. If $P=(1,0), Q=(-1,0)$ and $R=(2,0)$ are three given points, then the locus of a point *S* satisfying the relation $SQ^2+SR^2=2SP^2$ is

[IIT 1988]

(a) A straight line parallel to *x*-axis (b) A circle through origin

(c) A circle with centre at the origin (d) A straight line parallel to *y*-axis

153. The locus of a point which moves in such a way that its distance from (0, 0) is three times its distance from the *x*-axis, as given by

[MP PET 1993]

(a) $x^2-8y^2=0$ (b) $x^2+8y^2=0$ (c) $4x^2-y^2=0$ (d) $x^2-4y^2=0$

154. $A(a,0)$ and $B(-a,0)$ are two fixed points of triangle $ABC$. The vertex *C* moves in such a way that $\cot A+\cot B=\lambda$, where $\lambda$ is a constant. Then the locus of the point *C* is [MP PET 1981]

(a) $y\lambda=2a$ (b) $ya=2\lambda$ (c) $y=\lambda a$ (d) None of these

155. A line of fixed length $(a+b)$ moves so that its ends are always on two fixed perpendicular lines. The locus of the point which divides this line into portions of lengths *a* and *b* is

(a) A circle (b) An ellipse (c) A hyperbola (d) None of these

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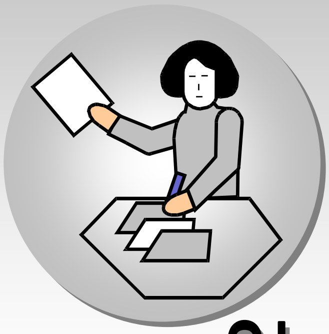

# Answer Sheet

Rectangular Cartesian Co-ordinates

*Assignment (Basic and Advance Level)*

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| a | c | c | c | d | a | b | d | b | d | b | b | a | d | c | c | b | c | c | c |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| c | c | c | b | a | b | b | b | a | b | b | c | d | b | c | d | d | b | c | b |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| b | c | b | b | c | c | a | c | a | c | c | d | c | a | a | a | c | c | a | b |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| c | a | c | a | a,c | b | c | b | a | d | a | b | b | b | a | c | a | a | a | b |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| a,c,d | b | c | d | a | b | d | b | a | a | d | d | a | a | a | a | c | d | c | c |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 | 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |
| b | b | b | b | a | c | c | a | a | a | b | c | d | d | d | a | a | d | c | a |
| 121 | 122 | 123 | 124 | 125 | 126 | 127 | 128 | 129 | 130 | 131 | 132 | 133 | 134 | 135 | 136 | 137 | 138 | 139 | 140 |
| c | d | b | c | b | b | a | a | b | a | a | a | d | b | b | b | a | b | b | d |

| 141 | 142 | 143 | 144 | 145 | 146 | 147 | 148 | 149 | 150 | 151 | 152 | 153 | 154 | 155 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| b | c | b | b | b | c | d | d | a | b | d | d | a | a | b |